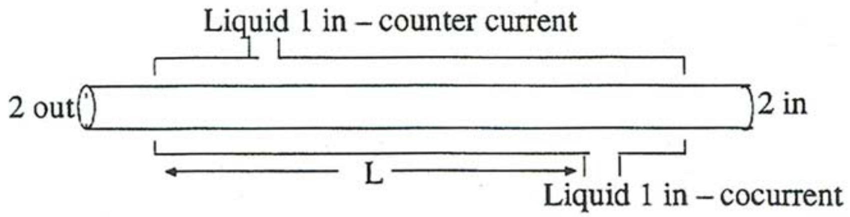
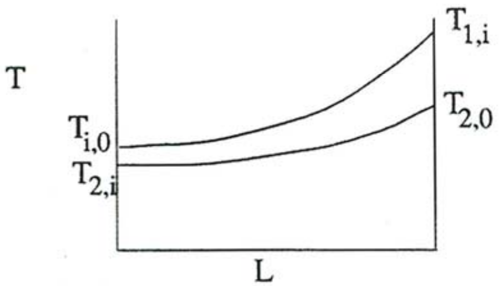
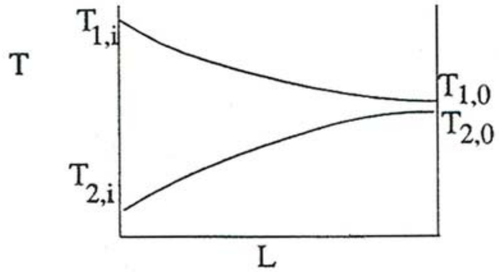
Heat Exchanges/Theory

The double-pipe heat exchanger





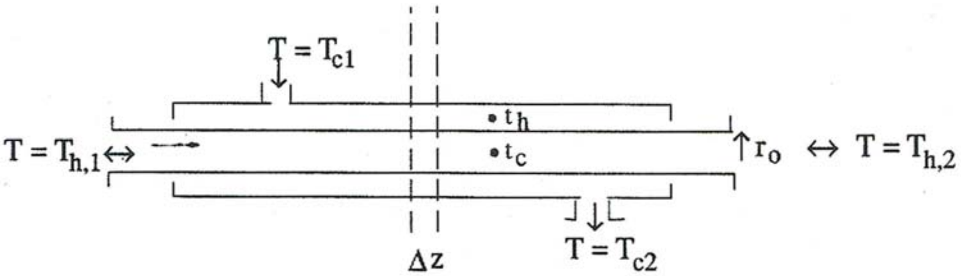
counter current



cocurrent

Call the hot stream $_h$ and the cold stream $_c$ and determine all coefficients w.r.t the outside diameter of inside pipe.

Usually, we need to figure out the required heat transfer area for a given job, given a flowrate and a temperature difference.



w – mass flow rate

Performing an energy balance on each stream separately, we get

$$dq = w_c \underset{\underset{\text{temp \& position}}{\uparrow}}{Cp_c} dt_c = -w_h \underset{\underset{\text{temp \&}\atop\text{position}}{\uparrow}}{Cp_h} dt_h = \underset{\underset{\text{temp \&}\atop\text{position}}{\uparrow}}{U_0\big|_z} \underset{\underset{\text{position}}{\uparrow}}{\left(t_h - t_c\right)\big|_z} \overset{\overset{2\pi r_0 dz}{\uparrow}}{dA_0}\big|_z$$

Usually, $w_c, Cp_c, w_h, Cp_h, U_0$ are temperature and position dependent.
To simplify, assume mass flow, and Cp_c, are independent of temperature hence, position.
Then integrate inside equations.

$$w_c\, Cp_c\left(T_{c,2} - T_{c,1}\right) = -w_h\, Cp_h\left(T_{h,2} - T_{h,1}\right) = Q$$

and

$$\begin{aligned} w_c\, Cp_c dt_c &= U_0\left(t_h - t_c\right) dA_0 \\ -w_h\, Cp_c dt_h &= U_0\left(t_h - t_c\right) dA_0 \\ \rightarrow \frac{dt_c}{t_h - t_c} &= \frac{U_0\, dA_0}{w_c\, Cp_c} \\ \rightarrow -\frac{dt_h}{t_h - t_c} &= \frac{U_0\, dA_0}{w_h\, Cp_h} \end{aligned}$$

if we add these two

$$-\left(\frac{d\left(t_h - t_c\right)}{\left(t_h - t_c\right)}\right) = U_0\, dA_0\left(\frac{1}{w_c\, Cp_c} + \frac{1}{w_h\, Cp_h}\right)$$

This equation relates the $\Delta t's$ to position
for U_0 independent of position, integrate from inlet (1) to outlet (2)

$$-\ell n\left(\frac{\left(t_{h,2} - t_{c,2}\right)}{\left(t_{h,1} - t_{c,1}\right)}\right) = U_0\, A_0\left(\frac{1}{w_c\, Cp_c} + \frac{1}{w_h\, Cp_h}\right)$$

$$\begin{aligned} \frac{1}{w_h\, Cp_h} &= \frac{t_{h,2} - t_{h,1}}{-Q} \\ \frac{1}{w_c\, Cp_c} &= \frac{t_{c,2} - t_{c,1}}{Q} \end{aligned}$$

Hence

$$\begin{aligned} -\ell n\left(\frac{\left(t_{h,2} - t_{c,2}\right)}{\left(t_{h,1} - t_{c,1}\right)}\right) &= \frac{U_0\, A_0}{Q}\left(\left(t_{c,2} - t_{c,2}\right) - \left(t_{h,1} - t_{h,1}\right)\right) \\ &= -\frac{U_0\, A_0}{Q}\left(\left(t_{h,2} - t_{c,1}\right) - \left(t_{h,2} - t_{c,1}\right)\right) \end{aligned}$$

$$U_0\, A_0\ \frac{\left(t_{h,2}-t_{c,2}\right)-\left(t_{h,1}-t_{c,1}\right)}{\ell n\dfrac{\left(t_{h,2}-t_{c,2}\right)}{\left(t_{h,1}-t_{c,1}\right)}}=Q=U_0\, A_0\ \Delta t_{\ell m}$$

$$\Delta t_{\ell m}=\ \frac{\left(t_{h,2}-t_{c,2}\right)-\left(t_{h,1}-t_{c,1}\right)}{\ell n\dfrac{\left(t_{h,2}-t_{c,2}\right)}{\left(t_{h,1}-t_{c,1}\right)}}$$

Linear Temperature Dependence of U_0 on Temperature

Let $U_0=a+b\ \Delta t$

and insert this in the balance equation

$$\begin{aligned}
w_c\, Cp_c dt_c &= U_0\left(t_h-t_c\right)dA_0\\
-w_h\, Cp_c dt_h &= U_0\left(t_h-t_c\right)dA_0\\
\frac{dt_c}{t_h-t_c} &= \frac{U_0}{w_c\, Cp_c}\, dA_0\\
-\frac{dt_c}{t_h-t_c} &= \frac{U_0}{w_h\, Cp_h}\, dA_0\\
-\frac{d\left(t_h-t_c\right)}{\left(t_h-t_c\right)} &= U_0\, dA_0\left(\frac{1}{w_c\, Cp_c}+\frac{1}{w_h\, Cp_h}\right)\\
\frac{d\left(t_h-t_c\right)}{U_0\left(t_h-t_c\right)} &= dA_0\underbrace{\left(\frac{1}{w_c\, Cp_c}+\frac{1}{w_h\, Cp_h}\right)}_{\dfrac{\Delta t_2-\Delta t_1}{Q}}
\end{aligned}$$

Let $U_0=a+b\left(t_h-t_c\right)$

$$t_h-t_c=\Delta t$$

$$\int_{\Delta t_1}^{\Delta t_2}\frac{d\left(\Delta t\right)}{\left(a+b\Delta t\right)\Delta t}=\frac{\Delta t_2-\Delta t_1}{Q}\ A_0$$

$$\left[\frac{1}{a}\right]\ell n\frac{\Delta t}{a+b\Delta t}\Big|_{\Delta t_1}^{\Delta t_2}\ =\ \left[\frac{1}{a}\right]\ell n\frac{\Delta t}{U_0}\Big|_{\Delta t_1}^{\Delta t_2}$$

$$-\frac{1}{a}\ell n\frac{\left(U\right)_1\left(\Delta t\right)_2}{\left(U\right)_2\left(\Delta t\right)_1}\ \text{ and}$$

$$\frac{1}{a}\ell n\ \frac{U_{0_1}\,\Delta t_2}{U_{0_2}\,\Delta t_1}\ =\ \frac{\Delta t_2-\Delta t_1}{Q}\ A_0$$

in which, $U_{0_1}=a+b\left(t_h-t_c\right)_1;U_{0_2}=a+b\left(t_h-t_c\right)_2$

$=a+b\ \Delta t_1$; $U_{0_2}=a+b\ \Delta t_2$

$$U_{0_1}\Delta t_2 = a\Delta t_2 + b\Delta t_1\Delta t_2$$

$$U_{0_2}\Delta t_1 = a\Delta t_1 + b\Delta t_1\Delta t_2$$

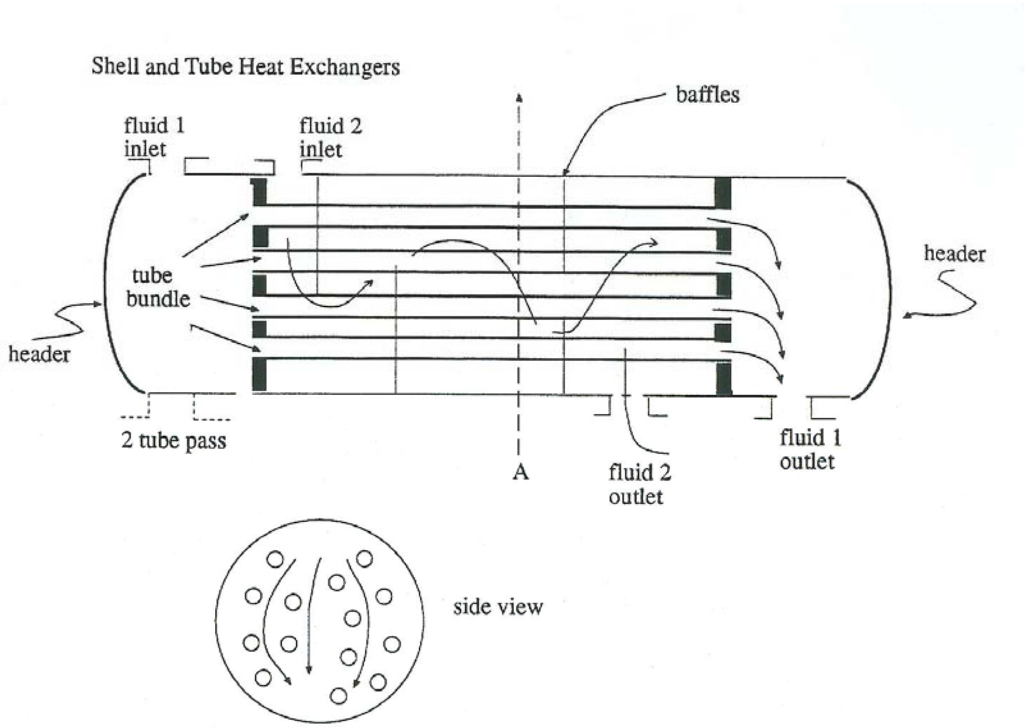
subtract

$$U_{0_1}\Delta t_2 - U_{0_2}\Delta t_1 = a\left(\Delta t_2 + \Delta t_1\right)$$

$$\frac{U_{0_1}\Delta t_2 - U_{0_2}\Delta t_1}{\Delta t_2 + \Delta t_1} = a$$

$$\frac{\cancel{\Delta t_2 - \Delta t_1}}{U_{o_1}\Delta t_2 - U_{o_2}\Delta t_1}\,\ell n\,\frac{U_{o_1}\Delta t_2}{U_{o_2}\Delta t_1} = \frac{\cancel{\Delta t_2 - \Delta t_1}}{Q}\,A_o$$

$$\boxed{Q = \frac{U_{0_1}\Delta t_2 - U_{0_2}\Delta t_1}{\ell n\dfrac{U_{0_1}\Delta t_2}{U_{0_2}\Delta t_1}}A_0}$$



shown – 1 shell pass – 1 tube concurrent heat exchanger

Benefits – cost, space, pressure drop

consists of bundle of tubes through which one fluid passes the other fluid flows through the shell. Baffles are used to direct the flow of one shell fluid to improve heat transfer.

1) If one fluid fouls — it should be routed through tubes.
2) If one fluid is at high pressure—should go through tubes to avoid cost of high pressure

shell.

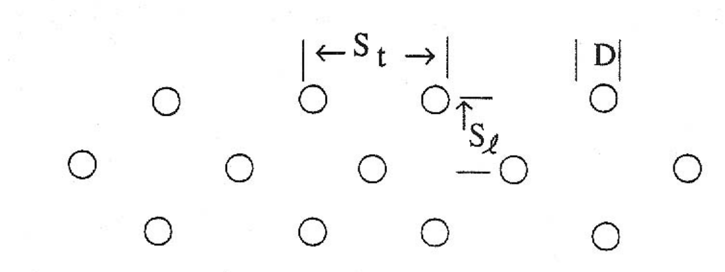
3) Corrosive material should go through tubes to avoid costs associated with shell construction.
4) More viscous fluid should be directed through shell ΔP is low.
5) If major **resist:lite** is in tube side, a 2 tube pass, one shell pass may be better.

In the 1-D analysis we get a similar result to the double-pipe exchanger

$$Q = U_0 A_0 \left[\frac{\left(t_{2,i} - t_{2,0}\right) - \left(t_{1,i} - t_{i,0}\right)}{\ell n \dfrac{t_{2,i} - t_{2,0}}{t_{1,i} - t_{1,0}}} \right]$$

where A_o is the total outside area of all the tubes, and U_o is the overall heat transfer coefficient.
How to evaluate U_o? — The tube side fluid is no problem, but to evaluate shell side.

Heat transfer coefficient for flow past tube bundles



s_t — transverse pitch
s_ℓ — longitudinal pitch

if $\gtrsim$ 10 rows of tubes in bundle and

$\gtrsim$ 10 tubes in each row, neglect entrance, end effects

and

$$Nu = F\left(\mathrm{Re}, \mathrm{Pr}, \mu_b / \mu_0, S_\ell, S_t\right)$$

$$S_\ell = s_\ell / D \quad S_t = s_t / D$$

experimental correlations show that

$$Nu = \left(0.4\, N_{\mathrm{Re}}^{1/2} + 0.2\, N_{\mathrm{Re}}^{2/3}\right) N_{\mathrm{Pr}}^{0.4} \left(\frac{\mu_b}{\mu_0}\right)^{.14}$$

$$N_{\mathrm{Re}} = \frac{D_p G}{\mu_b \left(1-\varepsilon\right)}$$

$$\mathrm{Pr} = \frac{Cp}{\mu_b k_b}$$

$$N_{Nu} = \frac{h\, D_p}{k_b}\left(\frac{\varepsilon}{\left(1-\varepsilon\right)}\right)$$

range $N_{\mathrm{Re}} = 1-40{,}000$ $\quad \dfrac{\mu_b}{\mu_0} \sim .1-5$

$\mathrm{Pr} \sim .7-800$ $\quad \varepsilon \sim .42-.65$

ε — void fraction $\equiv \dfrac{\text{volume of tubes}}{\text{total shell volume}}$

a_v — $\dfrac{\text{area for heat transfer}}{\text{unit volume}}$

D_p — hydraulic radius= 3/2 D tube

To evaluate h_{outside} using this tube bundle correlation, we need to estimate the average mass flow rate G.

If the diameter of the shell is D_s and the distance between baffles is L_b, then an average mass velocity is

$$G_{av} = \frac{4\dot{m}}{\pi\, D_s L_b}$$

$\dot{n}$ — mass flow rate of shell fluid and the Reynolds number is

$$\mathrm{Re} = \frac{D_p\ G_{av}}{\mu\left(1-\varepsilon\right)} = \frac{4\,D_p\ \dot{m}}{\pi\mu\, D_s L_b \left(1-\varepsilon\right)}$$

ε — void fraction and in terms of parameters

$$\mathrm{Re} = \frac{6\,D\,\dot{m}}{\pi\mu\, D_s L_b \left(1-\varepsilon\right)}$$

$$Nu = \left(.4\,N^{1/2} + .2\,N^{2/3}\right)\mathrm{Pr}^{4}\left(\frac{\mu_b}{\mu_0}\right)^{.14}$$

Fouling of heat transfer surfaces:
One of the most difficult aspects of heat exchange design is the reduction in heat transfer rates due to deposition of "scale"—insoluble inorganic compounds—on the tube wall. The magnitude depends on the nature and thickness. It is usually reported in terms of a film heat transfer coefficient for the scale which is referred to as the fouling factor

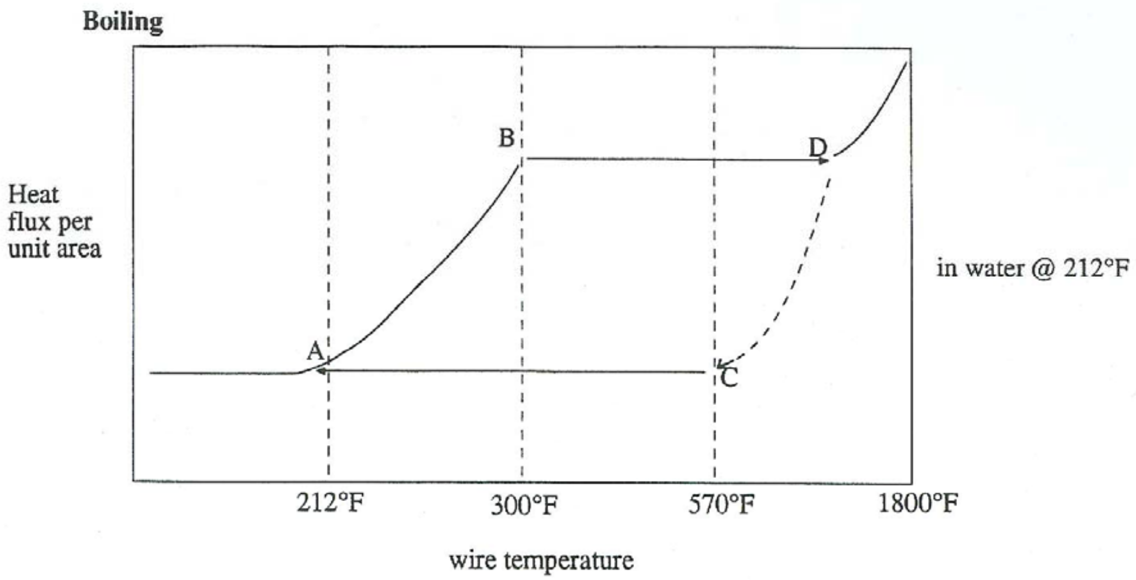
h_s = {film heat transfer for scale}

$\frac{1}{h_s} =$ fouling factor

to determine U_0, we assume all the resistance add in series:

$$\frac{1}{U_0} = \frac{1}{h_I} = \frac{D_0}{D_i} + \frac{1}{h_{s,i}}\left(\frac{D_0}{D_i}\right) + \frac{D_0}{2k}\ \ell n \frac{D_0}{D_i} + \frac{1}{h_{s,0}} + \frac{1}{h_{II}}$$

As the scale increases, the overall U_0 decreases, leading to poor performance and cleaning becomes necessary.



Observations

1. wire temperature rises smoothly to ~300°
2. temperature jumps to ~1800°
3. decrease heat flux to °570 wire temp at a much lower heat flux
4. jumps back down in temperature

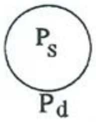
Explanation

Different types of boiling

Nucleate Boiling	**Transition**	**Film Boiling**
high heat transfer rate	intermediate unstable	low heat transfer rate

Nucleate boiling occurs directly at solid surface

Need to nucleate bubbles of gas somehow, but interface costs energy.

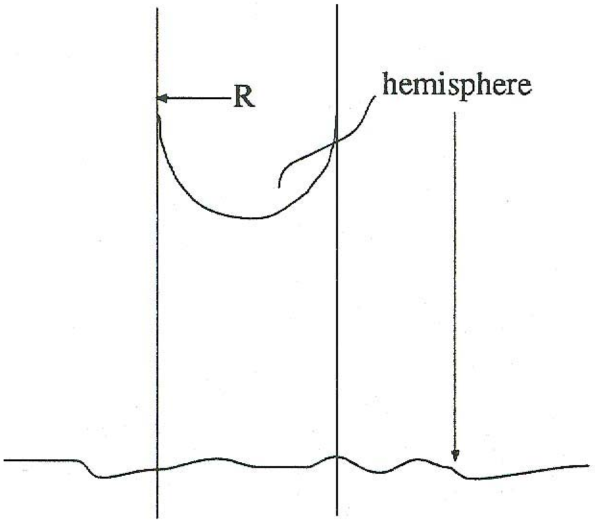


← surface tension A pressure drop exists across any interface due to surface tension.

$$\Delta P = 2H\sigma = \frac{2\sigma}{R}$$

$H =$ mean curvature of interface $= \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

capillary rise in a tube: $\rho_\ell gh - \rho_r gh = \frac{2\sigma}{R}$



Note that as $R \to 0,\ \Delta P \to \infty$ and to achieve the excess pressure, we need to achieve superheat.

extreme → case	homogeneous nucleation — bubbles of gas form spontaneously by molecular aggregation
most → common	heterogeneous nucleation — bubbles form at existing gas pockets, dust, dirt, etc.

Surface irregularities also are heterogeneous nuclei

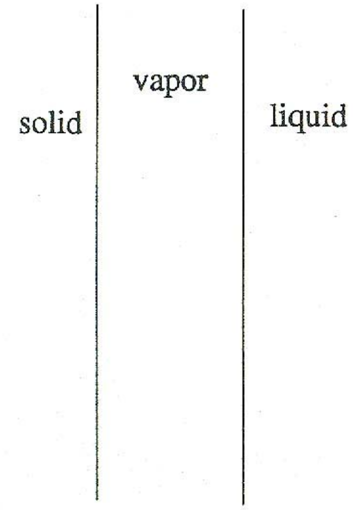
Film Boiling

Heat flux causes sufficient boiling that a vapor film covers surface, slows down heat transfer.

$$k_{water} = .35 \frac{Btu}{hr\ \text{ft}^2\ {}^\circ F}$$

$$k_{vapor} = .0183\ \frac{Btu}{hr\ \text{ft}^2\ {}^\circ F}$$

at least a factor of 20!



Boiling Summary

1) Heat flux is highest when change of phase occurs
2) Nucleate Boiling - highest rates, bubbles form directly at surface
3) Film Boiling - intermediate rates, bubbles form at vapor-liquid interface

Condensation

Condensation is used to heat solids in the same way boiling is often used to cool solids.

Similarly also, these are two types of condensation:

1) Dropwise ↔ Nucleate
2) Film ↔ Film

1) Dropwise —condensate doesn’t wet surface. forms drops
2) Film —condensate wets surface, forms liquid film